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Errata

Simultaneous Design of Active Vibration Control and Passive Viscous Damping

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THE author has identified several errors in this paper. They are as follows:

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Eq. (28) should read as follows:

$$C_{ii} = \max\left(\frac{\sum_{j=1}^{n} \Phi_{ij} W_{ij}}{\sum_{j=1}^{n} \Phi_{ij}^2}, 0\right); \qquad C_{ij} = 0 \text{ for } i \neq j \qquad (28)$$

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The last paragraph of the Appendix (which begins with "In the case where") should be replaced with the following:

The last sentence of the following theorem will be used to prove that C as defined in Eq. (28) minimizes the norm $\|C\Phi - W\|_F$ over the set of positive semidefinite diagonal matrices:

Theorem.[16] Let $\boldsymbol{x}$ be a vector in a Hilbert space H and let K be a closed convex subset of H. Then there is a unique vector $\boldsymbol{k}_0 \in K$ such that

$$\|x - \boldsymbol{k}_0\| \leq \|x - \boldsymbol{k}\|$$

for all $\boldsymbol{k} \in K$. Furthermore, a necessary and sufficient condition that $\boldsymbol{k}_0$ be the unique minimizing vector is that $(\boldsymbol{x} - \boldsymbol{k}_0, \boldsymbol{k} - \boldsymbol{k}_0) \leq 0$ for all $\boldsymbol{k} \in K$.

Let H be the space of all $m \times n$ matrices. The following functional satisfies the four requirements of an inner product and will be defined as the inner product on H:

$$(y, z) = \sum_{i=1}^{m} \sum_{j=1}^{n} y_{ij} z_{ij} \;\forall\; y, z \in H$$

Let $\Omega = \{E \in R^{m \times m} | E$ is positive semidefinite diagonal$\}$ and let $K = \{k | k = E\Phi$ where $E \in \Omega\}$. Because Ω is convex, K is convex also $(0 < \alpha < 1)$:

$$\alpha E_1 + (1 - \alpha) E_2 \in \Omega$$

therefore

$$[\alpha E_1 + (1 - \alpha) E_2]\Phi = \alpha E_1 \Phi + (1 - \alpha) E_2 \Phi \in K$$

Because K is equal to its closure, it is closed.

The inner product of $W - C\Phi$ with $E\Phi - C\Phi$ is as follows.

$$(W - C\Phi, E\Phi - C\Phi) = \sum_{i=1}^{m} \left[C_{ii}^2 \sum_{j=1}^{n} \Phi_{ij}^2 - C_{ii} \sum_{j=1}^{n} \Phi_{ij} W_{ij} - C_{ii} E_{ii} \sum_{j=1}^{n} \Phi_{ij}^2 + E_{ii} \sum_{j=1}^{n} \Phi_{ij} W_{ij} \right]$$

For each i such that $\sum_{j=1}^{n} \Phi_{ij} W_{ij} > 0$, the corresponding term in the summation above is zero:

$$\left(\frac{\sum_{j=1}^{n} \Phi_{ij} W_{ij}}{\sum_{j=1}^{n} \Phi_{ij}^2}\right)^2 \sum_{j=1}^{n} \Phi_{ij}^2 - \left(\frac{\sum_{j=1}^{n} \Phi_{ij} W_{ij}}{\sum_{j=1}^{n} \Phi_{ij}^2}\right) \left[\sum_{j=1}^{n} \Phi_{ij} W_{ij} - E_{ii} \sum_{j=1}^{n} \Phi_{ij}^2\right] + E_{ii} \sum_{j=1}^{n} \Phi_{ij} W_{ij} = 0$$

For each i such that $\sum_{j=1}^{n} \Phi_{ij} W_{ij} \leq 0$, the corresponding term in the summation over i in the inner product is negative for all E:

$$E_{ii} \sum_{j=1}^{n} \Phi_{ij} W_{ij} \leq 0 \quad \text{for} \quad \sum_{j=1}^{n} \Phi_{ij} W_{ij} \leq 0 \text{ and } E \in K$$

Hence, the summation over i in the inner product reduces to a sum of zeros and negative terms, and is therefore less than (or equal to) zero for all $E \in K$. This proves that C as defined in Eq. (28) minimizes the Frobenius norm $\|C\Phi - W\|_F$ over the set of positive semidefinite diagonal matrices.